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Methodology Article

Contraction, Lebesgue and Common Fixed Point Property of Fuzzy Metric Spaces

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Abstract: In this paper, we using contraction and contraction functions in complete fuzzy metric space and establish sequential characterization properties of Lebesgue fuzzy metric space and common fixed on it. Then first introduce a new type of Lebesgue fuzzy metric space, which is generalization of fuzzy metric space, second we study the topological properties of Lebesgue fuzzy metric space, third a relation between Lebesgue and weak G-complete, compact fuzzy metrics and Lebesgue integral mappings finally established characterization properties on it. We prove the existence of common fixed point and contraction mapping in fuzzy metric space using the property of Lebesgue fuzzy metric space and integral type of mappings. On the basis of these properties we are getting common fixed point of two mappings, three mappings and four mappings in a easy way as compared to old method like Banach contraction fixed point. Also coincidence fixed point theorem for two mapping, three mappings and four mappings using Lebesgue fuzzy metric space and integral type of mappings. Also contraction mappings property in fuzzy metric space is helpful to determine common fixed point in Lebesgue fuzzy metric space. We also discuss the Lebesgue property of several well-known fuzzy metric spaces in this paper and conclude uniqueness of common fixed point.

Keywords: Fuzzy Metric Space, Completeness, Continuity, Contraction Function, Fixed Point, Lebesgue Property, G-complete

1. Introduction

In 1965 Lotfi Aliasker Zadeh [15] introduces the concept of fuzzy sets since one of the important problems is to obtain an adequate notion of fuzzy metric space. In 1975 Kramosil and Michalek [9] reformulated successfully the notion probabilistic metric space introduced by Menger in 1942, in fuzzy context. After that, George and Veeramani [2] modified the concept of fuzzy metric space. Heipern [6] in 1981 first proved a fixed point theorem for fuzzy functions. Also Mariusz Grabeic [3] in 1988 proved the contraction principle in the setting of the fuzzy metric spaces. Moreover a George and Prasanthini Veeramani [2] in1994 modified the notion of fuzzy metric spaces with the help of t-norm.

Also we know that every fuzzy metric gives rise to a metrizable topology that allowed the researchers to adopt several concepts from metric spaces in this fuzzy setting. Valentin Gregori, Salvador Romaguera and Almanzor sapena [6] introduced a notion similar to the Lebesgue number in fuzzy metric spaces.

In the theory of metric spaces, the Lebesgue number states that every open cover U of a compact metric space (X, d) corresponds to a positive number δ such that any subset of X having diameter less than δ gets contained in some member of U. This is called a Lebesgue number for U. The property of having such positive real numbers for every open cover is called the Lebesgue property for metric spaces. In 2012 Geogori, Romaguera and Sapena [6] gave a satisfactory extension to the notion of Lebesgue property for fuzzy metric spaces and characterized Uniform continuity, equinormality and uniformility. In 2002 Alberto Branciari [1] introduce contractive condition of integral type. In 2017 Valentin Gregori, Juan Jose. Minana, Almanzor Sapena [4] proved compatible fuzzy metric spaces. In 2018 Valentin Gregori, Juan Jose Minana, Almanzor Sapena [5] gave Banach contradiction principle in fuzzy metric spaces. In 1981 Shandong Heipern [7] gave fuzzy mapping in fixed point theorem. In 2014 Nawab Hussain, Marwan Kutbi and P. Salini [8] proved fixed point in α-complete metric space. In 2001 Almanzor Sapena [11] contribute to the study of fuzzy metric spaces. In 2003 Reed Vasuki, Prasanthini Veeramani [12] gave fixed point theorems and Cauchy sequences in fuzzy metric spaces. In 2001 Prasanthini Veeramani [13] gave Best approximation in fuzzy metric spaces. In 2014 Binod Chandra Tripathi, Szemeredi Paul and Nand Ram Das [14] proved fixed point theorem in a generalized fuzzy metric space. In 2004 Jose Rodriguez-Lopez, Salvador Romaguera [10] gave the Hausdorff Fuzzy metric space on compact sets.

2. Preliminaries

Definition 2.1: Let * be a binary operation on I=[0,1] which is associative, commutative and continuous on I × I then $*$ is said to be a continuoust-norm, if

(a) $a * 1 = a \ \forall\ a \in [0, 1]$

(b) $a \leq b,\ c \leq d) \Rightarrow\ a * c \leq b * d\ \ \forall\ a.b.c.d \in [0, 1]$.

Definition 2.2: Given a non-empty set X, a continuous t-norm $*$ and a mapping M: $X\times X\times(0, \infty) \to [0, 1]$, the ordered pair (M, $*$) is said to be a fuzzy metric on X if for all x, y in X and t >0, the following conditions hold:

(a) $M(x, y, t) > 0$; $M(x, y, t)=1 \Leftrightarrow x=y$

(b) $M(x, y, t)=M(y, x, t)$

(c) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$

(d) $M(x, y,.): (0, \infty) \to [0, 1]$ is continuous then (X, M, $*$) is fuzzy metric space.

Result: Let (X, M, $*$) be a fuzzy metric space, then $B_M(x, r, t)$: $x\in X$, $r\in (0, 1)$, t >0, where $B_M(x, r, t)=y\in X, M(x, y, t)>1-t$ forms a base for some topology T_M on X.

Definition 2.3: T_M is called the toplogy induced by (M, $*$).

Definition 2.4: Let (X, d) be a metric space. If Md: $X\times X\times (0, \infty) \to [0, 1]$ is defined for all $x,y\in X$ and t >0 by

$$M_d(x, y, t)=\frac{t}{t+d(x,y)}$$

then (Md, '.') is called standard fuzzy metric induced by d.

Definition 2.5: A sequence (x_n) in a fuzzy metric space (X, M, $*$) is said tobe Cauchy sequence if for $\varepsilon \in (0, 1)$ and t >0 there $\exists$ $k\in N$ such that

$M(x_m, x_n, t) > 1-\varepsilon\ \forall\ m, n \geq k$.

A sequence (x_n) in a fuzzy metric space (X, M, $*$) converges to $x\in X \Leftrightarrow \lim\limits_{n\to\infty} M(x_n,x, t)=1$.

Definition 2.6: A sequence (x_n) in a fuzzy metric space (X, M, $*$) is said to be Pseudo Cauchy if given $\varepsilon \in (0, 1)$, t >0 and $k\in N$ there exists j, $n(> k)\in N$ with $j \neq n$ such that $M(x_j, x_n, t) > 1-\varepsilon$.

Definition 2.7: A sequence (x_n) in a fuzzy metric space (X, M, $*$) is said to be G-Cauchy sequence if for each t >0 and $p \in N$, $\lim\limits_{n\to\infty} M(x_n,x_{n+p}, t)=1$.

Definition 2.8: A fuzzy metric space (X, M, $*$) is said to be

(1) Weak G-complete if every G-Cauchy sequence in X has a limit in it.

(2) G-complete if every G-Cauchy sequence in X converges in it.

Definition 2.9: Let f: $X \to X$ and α: $X\times X \to [0, \infty)$ be two function then f is said to be α-admissible function if

$\alpha(x; y)\geq 1 \Rightarrow \alpha(f(x); f(y)) \geq 1\ \forall\ x, y\in X$.

Definition 2.10: Let (X, M, $*$) be a fuzzy metric space and α, β: $X\times X \to [0, \infty)$ then X is said to be α-β complete if every Cauchy sequence x_n with

$\alpha(x_n,x_{n+1}) \geq\beta(x_n,x_{n+1})\ \forall\ n\in N$ converge in X.

Definition 2.11: Let (X, M, $*$) be a fuzzy metric space and α, β: $X\times X \to [0, \infty)$ and f: $X \to X$, then f is said to be α-β continuous function on X if for $x\in X$ and (x_n) be sequence in X with $x_n \to x$ as $n \to \infty$ such that

$\alpha(x_n, x_{n+1})\geq\beta(x_n, x_{n+1})\ \forall\ n\in N \Rightarrow\ f(x_n) \to f(x)$.

Definition 2.12: Let (X, M, $*$) be a fuzzy metric space and α, β: $X\times X \to [0, \infty)$ and f: $X \to X$, then f is said to be $\alpha-\beta-\varphi$-contraction function on X if

$\alpha(x; y)\geq\beta(x; y) \Rightarrow M(f(x), f(y), t)\geq \varphi M(x, y)$.

Definition 2.13: A fuzzy metric space (X, M, $*$) is said to have the Lebesgue property if given an open cover G of (X, T_M), there exists $r\in (0; 1)$, t >0 such that $B_M(x, r, t)$: $x\in X$ defines G.

Definition 2.14: A metric space (X, d) is Lebesgue iff (X, M_d) is Lebesgue.

Definition 2.15: A fuzzy metric space (X, M, $*$) is said to be equinormal if for given non-empty closed subset B and C of (X, T_M) with $B \cap C=\varphi$, there exists s>0 such that sup(M(b, c, s): $b\in B$; $c\in C$) <1.

3. Main Result

Theorem 3.1: Let (X, M, $*$) be a fuzzy metric space and A, B: $X \to X$, suppose that α, β: $X\times X \to [0, \infty)$ are two function such that

(1) (X, M, $*$) is an α-β fuzzy complete metric space

(2) A, B is α-admissible function with respect to β

(3) A, B is α-β-φ-contraction function on X.

(4) A, B is α-β continuous function

(5) If M(x, y)=min(M(x, Ax, t), M(By, Ax, t), M(y, By, t), M(x, y, t)) then there exist $x_0\in X$ such that $\alpha(x_0, f(x_0))\geq\beta(x_0, f(x_0))$ then A, B has a common fixed point in X, if f is A or B.

Proof: Given(X, M, $*$) be a fuzzy metric space then there exist $x_0\in X$ such that

$\alpha(x_0, f(x_0))\geq\beta(x_0, f(x_0)))$.

Define a sequence $(xn)\in X$ such that $x_n=f(x_{n-1})\ \forall\ n\in N$.

If $x_n=x_{n+1}$ then $x=x_n$ is a common fixed point of A, B.

Suppose $x_n\neq x_{n+1}$, but f is α-admissible such that

$\alpha(x_0, f(x_0))\geq\beta(x_0, f(x_0))$

$\Rightarrow \alpha(x_1, x_2))\geq\alpha(f(x_0), f(x_1))\geq\beta(f(x_0), f(x_1))$

$\geq\beta(x_1, x_2))$

....

$M(x_n, x_{n+1}, t)=M(f(x_{n-1}), f(x_n), t)\geq\alpha(M(x_{n-1}, x_n)$

Since M(x, y)=min(M(x, Ax, t), M(By, Ax, t), M(y, By, t), M(x, y, t))

Consider $Ax_n=Bx_n=x_{n+1}$

Then $M(x_{n-1}, x_n)=\min(M(x_n, x_{n+1}, t), M(x_{n+1}, x_{n+2}, t), M(x_{n+1}, x_{n+2}, t), M(x_n, x_{n+1}, t))$

$\Rightarrow M(x_n, x_{n+1}, t) \geq \alpha(M(x_{n-1}, x_n) \geq \alpha \min M(x_n, x_{n+1}, t)$

which is contradiction

Therefore $x_n=x_{n+1}$

Hence

$M(x_n, x_{n+1}, t)=\varphi M(x_{n-1}, x_n, t)=\varphi^2 M(x_{n-2}, x_{n-1}, t)$

....

Then

$M(x_n, x_{n+1}, t)=\varphi^n M(x_0, x_1, t)$

Therefore

$\lim_{n\to\infty} M(x_n, x_{n+1}, t)=1$

So A, B has a common fixed point as $x_n=x_{n+1}=x$

In other words $f(x)=x$ is a fixed point.

Suppose y is a another fixed point of f such that $f(y)=y$. $M(x, y, t)=M(f(x), f(y), t) \geq \varphi M(x, y)$

Hence $x=y$.

Theorem 3.2: Let (X, M, $*$) be a fuzzy metric space and A, B, T: $X \to X$, suppose that α, β: $X\times X \to [0, \infty)$ are two function such that

(1) (X, M, $*$) is an α-β fuzzy complete metric space

(2) A, B, T is α-admissible function with respect to β

(3) A, B, T is α-β-φ-contraction function on X.

(4) A, B, T is α-β continuous function

(5) If $M(x, y)=\min(M(y, Ty, t), M(Ax, Ty, t), M(Tx, By, t))$ then there exist

$x_0 \in X$ such that $\alpha(x_0, f(x_0)) \geq \beta(x_0, f(x_0))$ then A, B, T has a common fixed point in X, if f is A, B or T.

Proof: Given (X, M, $*$) be a fuzzy metric space then there exist $x_0 \in X$ such that

$\alpha(x_0, f(x_0)) \geq \beta(x_0, f(x_0))$

Define a sequence $(x_n) \in X$ such that

$x_n=f(x_{n-1})$ $\forall n \in N$.

If $x_n=x_{n+1}$ then $x=x_n$ is a common fixed point of A, B, T.

Suppose $x_n \neq x_{n+1}$, but f is α-admissible such that

$\alpha(x_0, f(x_0)) \geq \beta(x_0, f(x_0))$

$\Rightarrow$ $\alpha(x_1, x_2)) \geq \alpha(f(x_0), f(x_1)) \geq \beta(f(x_0), f(x_1))$

$\geq \beta(x_1, x_2))$

....

$M(x_n, x_{n+1}, t)=M(f(x_{n-1}), f(x_n), t) \geq \alpha(M(x_{n-1}, x_n)$

Since $M(x, y)=\min(M(y, Ty, t), M(Ax, Ty, t), M(Tx, By, t))$

Consider $Ax_n=Bx_n=Tx_n=x_{n+1}$

Then $M(x_{n-1}, x_n)=\min(M(x_n, x_{n+1}, t), M(x_n, x_{n+1}, t), M(x_n, x_{n+1}, t))$

$\Rightarrow M(x_n, x_{n+1}, t) \geq \alpha(M(x_{n-1}, x_n) \geq \alpha \min M(x_n, x_{n+1}, t)$

which is contradiction.

Therefore $x_n=x_{n+1}$

Hence

$M(x_n, x_{n+1}, t)=\varphi M(x_{n-1}, x_n, t)=\varphi^2 M(x_{n-2}, x_{n-1}, t)$

....

Then

$M(x_n, x_{n+1}, t)=\varphi^n M(x_0, x_1, t)$

Therefore

$\lim_{n\to\infty} M(x_n, x_{n+1}, t)=1$.

So A, B, T has a common fixed point as $x_n=x_{n+1}=x$.

In other words $f(x)=x$ is a fixed point.

Suppose y is a another fixed point of f such that $f(y)=y$, $M(x, y, t)=M(f(x), f(y), t) \geq \varphi M(x, y)$

Hence $x=y$.

Theorem 3.3: Let (X, M, $*$) be a fuzzy metric space and A, B, S, T: $X \to X$, suppose that α, β: $X\times X \to [0, \infty)$ are two function such that

(1) (X, M, $*$) is an α-β fuzzy complete metric space

(2) A, B, S, T is α-admissible function with respect to β

(3) A, B, S, T is α-β-φ-contraction function on X.

(4) A, B, S, T is α-β continuous function

(5) If $M(x, y)=\min M(Sx, Ty, t), M(Ax, Ty, t), M(Sx, By, t)$ then there exist $x_0 \in X$ such that $\alpha(x_0, f(x_0)) \geq \beta(x_0, f(x_0))$ then A, B, S, T has a common fixed point in X, if f is A, B, S or T.

Proof: Given (X, M, $*$) be a fuzzy metric space then there exist $x_0 \in X$ such that

$\alpha(x_0, f(x_0)) \geq \beta(x_0, f(x_0))$.

Define a sequence $(x_n) \in X$ such that

$x_n=f(x_{n-1})$ $\forall n \in N$

If $x_n=x_{n+1}$ then $x=x_n$ is a common fixed point of A, B, S, T.

Suppose $x_n \neq x_{n+1}$, but f is α-admissible such that

$\alpha(x_0, f(x_0)) \geq \beta(x_0, f(x_0))$

$\Rightarrow$ $\alpha(x_1, x_2)) \geq \alpha(f(x_0), f(x_1)) \geq \beta(f(x_0), f(x_1))$

$\geq \beta(x_1, x_2))$

....

$M(x_n, x_{n+1}, t)=M(f(x_{n-1}), f(x_n), t) \geq \alpha(M(x_{n-1}, x_n)$

Since $M(x, y)=\min M(Sx, Ty, t), M(Ax, Ty, t), M(Sx, By, t)$

Consider $Ax_n=Bx_n=Sx_n=Tx_n=x_{n+1}$

Then $M(x_{n-1}, x_n)=\min(M(x_n, x_{n+1}, t), M(x_n, x_{n+1}, t), M(x_n, x_{n+1}, t))$

$\Rightarrow M(x_n, x_{n+1}, t) \geq \alpha(M(x_{n-1}, x_n) \geq \alpha \min M(x_n, x_{n+1}, t)$

which is contradiction

Therefore $x_n=x_{n+1}$

Hence

$M(x_n, x_{n+1}, t)=\varphi M(x_{n-1}, x_n, t)=\varphi^2 M(x_{n-2}, x_{n-1}, t)$

....

Then

$M(x_n, x_{n+1}, t)=\varphi^n M(x_0, x_1, t)$.

Therefore

$\lim_{n\to\infty} M(x_n, x_{n+1}, t)=1$

So A, B, S, T has a common fixed point as $x_n=x_{n+1}=x$

In other words $f(x)=x$ is a fixed point

Suppose y is a another fixed point of f such that $f(y)=y$, $M(x, y, t)=M(f(x), f(y), t) \geq \varphi M(x, y)$

Hence $x=y$.

Theorem 3.4: If (X, M, $*$) be a fuzzy complete metric space, where $*$ is a continuous t-norm. If the mapping

f: $X \to X$ is φ-contraction, then f has a unique fixed point $x \in X$ and for all $x_0 \in X$, $\lim_{n\to\infty} f^n(x_0)=x$.

Proof: Consider x0 be an arbitrary point in X then define a sequence (xn) by xn+1= $f^n(x_0)$ $\forall n \geq 0$

Suppose f is a φ-contraction, then

$M(xn, xn, \varphi(t))=M(f(x_{n-1}), f(x_{n-1}), \varphi(t))$

$\geq$ M(x_{n-1}, x_{n-1}, t)

So (x_n) is a Cauchy sequence in X. Since X is complete then there exists x$\in$ X such that

$\lim_{n\to\infty} x_n$ =x $\forall$ t >0

$\lim_{n\to\infty}$ M(x_n, x_n, t)=1

Now, we will prove that x is a fixed point of f, for t>0 there exist r $\geq$ t such that φ(r) < t then

M(x, x, t)$\geq *$ (M(x, x, $\frac{t-\varphi(r)}{2}$), M(x, x, $\frac{t-\varphi(r)}{2}$),

M(f(x), f(x), φ(r)), i.e., M(x, x, t)$\geq$ $*$ ((M(x, x, $\frac{t-\varphi(r)}{2}$), M(x, x, $\frac{t-\varphi(r)}{2}$), M(x, x, r)).

Since $\lim_{n\to\infty} x_n$ =x= $\lim_{n\to\infty} x_{n+1}$ and $*$ is a continuous then M(f(x), f(x), t)$\geq$ M(1, 1, 1)=1. So f(x)=x that x is a fixed point, we can easily show that x is unique.

Theorem 3.5: A Lebesgue fuzzy metric space (X, M, $*$) is G-complete.

Proof: Given (X, M, $*$) be a Lebesgue fuzzy metric space and (x_n)$\in$ X then (x_n) is a G-Cauchy sequence in X. If (x_n) has a constant subsequence, then it has a limit point in X.

If (x_n) has not a constant subsequence then

(x_n)=(x_1, x_2, x_3, ………… x_n, …..).

Now we consider $x_1 \neq x_2 \neq$then

M(x_1, x_2, $\frac{1}{2}$)>1-$\frac{1}{2}$

$\Rightarrow$ M(x_2, x_3, $\frac{1}{3}$)>1-$\frac{1}{3}$>1-$\frac{1}{2}$

……….

M(x_{2n}, x_{2n+1}, $\frac{1}{2n+1}$)>1-$\frac{1}{2n+1}$>1-$\frac{1}{n}$

Therefore

M(x_{2n}, x_{2n+1}, t) > M(x_{2n}, x_{2n+1}, $\frac{1}{n}$)>1-$\frac{1}{n}$$\geq$1-$\varepsilon$.

Consequently (x_n) is a fuzzy Cauchy sequence and has a limit point in X, hence this complete the proof.

Theorem 3.6: A Lebesgue fuzzy metric space (X, M, $*$) is G-Hausdorff.

Proof: Given that (X, M, $*$) be a Lebesgue fuzzy metrc space then there exists x and y be two distinct points of X, such that 0 < M(x, x, t) <1 for some t>0, suppose r=M(x, x, t) where 0 < r <1, then there exists open balls

M1(x, 1-r, $\frac{t}{2}$) and M2(y, 1-r,$\frac{t}{2}$) are open ball such that M_1(x, 1-r, $\frac{t}{2}$)$\cap$ M_2(y, 1-r,$\frac{t}{2}$)=φ

If above condition not hold, then we consider

z$\in$ M_1(x, 1-r, $\frac{t}{2}$)$\cap$$M_2$(y, 1-r,$\frac{t}{2}$) such that r=M(x, x, t)

$\geq$ M(x, x, $\frac{t}{2}$) $\cap$M(y, y,$\frac{t}{2}$)>r

which is a contradiction.

Therefore (X, M, $*$) is G-Hausdorff.

Corollary 1: If (X, M, $*$) be a fuzzy metric space then (X, M, $*$) is Lebesgue iff every fuzzy Cauchy sequence in (X, M, $*$) having distinct terms has a limit point in (X, T_M).

Corollary 2: (N, M, $*$) is Lebesgue fuzzy metric space.

Theorem 3.7: If (X, M, $*$) be a fuzzy complete metric space, k$\in$ (0, 1), and A, B: X$\to$ X be a mapping such that for each x, y$\in$ X

$$\int_0^{M(Ax,By,t)} \varphi(s)ds \leq k \int_0^{M(x,y,t)} \varphi(s)ds$$

where φ:[0, ∞)$\to$ [0, ∞) is Lebesgue integrable mapping which is summable on each compact subset of [0, ∞) and

M(Ax, By, t)=min(M(x, Ax, t), M(By, Ax, t), M(y, By, t), M(x, y, t)) such that for each ε>0

$$\int_0^{\varepsilon} \varphi(s)ds > 0.$$

Then A, B has a unique fixed point z$\in$ X,

$\lim_{n\to\infty} A^n x$ =z= $\lim_{n\to\infty} B^n x$.

Proof: We have given that

M(Ax, By, t)=min(M(x, Ax, t), M(By, Ax, t), M(y, By, t), M(x, y, t))

If x_n=x_{n+1}=z then z is a common fixed point, if $x_n \neq x_{n+1}$ then put x=x_n and y=x_{n+1} and Ax_n=Bx_n=x_{n+1} then

M(Ax, By, t)=min(M(x, Ax, t), M(By, Ax, t), M(y, By, t), M(x, y, t))

reduces to

M(x_{n+1}, x_{n+2}, t)=min(M(x_n, x_{n+1}, t), M(x_{n+1}, x_{n+2}, t), M(x_{n+1}, xn_{+2}, t), M(x_n, x_{n+1}, t))

$\Rightarrow$ M(x_{n+1}, x_{n+2}, t)=M(x_n, x_{n+1}, t)

which is contradiction

Therefore x_n=x_{n+1}

Hence

M(x_n, x_{n+1}, t)=kM(x_{n-1}, x_n, t)=k^2M(x_{n-2}, x_{n-1}, t)

……….

Then

M(x_n, x_{n+1}, t)=k^nM(x_0, x_1, t)

Therefore

$\lim_{n\to\infty}$ M(x_n, x_{n+1}, t)=1

So A, B has a common fixed point as x_n=x_{n+1}=z

In other words f(x)=x is a fixed point.

Suppose y is a another fixed point of f such that f(y)=y,

M(x, y, t)=M(f(x), f(y), t) $\geq$ kM(x, y)

Hence x=y.

Example: (G-complete, non-Lebesgue metric space) Consider X=(n: n$\in$ N)$\cup$(n +$\frac{1}{n}$: n$\in$ N) and d-usual metric on R to X$\times$ X.

Solution: As (X, d) metric space which is not Lebesgue then T_d is discrete topology on X. Thus

{(x): x$\in$ X} is a open cover of X without any Lebesgue number, So

(X,d) is not Lebesgue metric space, but (X, d) is G-complete metric space.

If possible $d(x_{n+1}; x_n) < \frac{1}{2}$ for $n \in N$ which contradict the properties of G-Cauchy sequence, Hence (X, d) is G-complete.

4. Conclusion

In this paper, We have shows that the fuzzy metric space of two mappings, three mappings and four mapping of itself has common fixed point by using the Lebesgue property. Similarly integral type of mappings., contraction mappings and compactness property has a common fixed point of two mappings, three mappings and four mappings. Also we conclude to determine uniquenee of common fixed point, Also we have common fixed point by Lebesgue property which is applied in fixed point theory and non-expansive fixed point theory.

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